**Question 3**

**Part a**

i) {-3, 3}

ii) {0, 1}

iii) {<1,a>, <1,b>, <1,c>, <2,a>, <2,b>, <2,c>}

iv) {<0,a>, <0,b>, <0,c>, <-3,a>, <-3,b>, <-3,c>, <3,a>, <3,b>, <3,c>}

**Part b**

i) Need to show reflexivity, symmetry and transitivity. Can be done easily with known properties of ‘=’

ii) The function performs x%3

N/R = {{3x|x in N}, {3x + 1| x in N}, {3x + 2| x in N}}

**Part c**

i) Bijection: g(x) = x + 1

ii) Injective but not surjective (image set is only positive)

ii) Surjective but not injective (Points of inflection lead to duplication)

**Part d**

A - B = {} so A must be a subset of B

A and B are finite and there is a surjection from A onto B so they must have the same cardinality

Therefore A = B

**4.a.i.**

example solution:





**4.a.ii.**

4 automorphisms

Node B can be mapped to 2 nodes (itself and E). Once B has been fixed, node A can be mapped to 2 nodes (itself and C). Once A has been fixed, no other mappings can be made.

Therefore number of automorphisms = 2 x 2 = 4

Alternatively, see that A can be mapped to 4 nodes (itself, C, D and F). Once A has been fixed, no other mappings can be made -> 4 automorphisms.

**4.b.i.**

A connected graph has an Euler path iff it has 0 or 2 odd nodes.

**4.b.ii**

We know the graph has either 0 or 2 odd nodes, from (i.), so lets do it case by case:

**Graph has 0 odd deg. Nodes**

All nodes have even deg. -> there exists a cycle through the entire graph, as any time you go into a node, you can always go out -> Euler Path.

**Graph has 2 odd deg. Nodes**

Like above, but after creating a path which goes through all even deg. Nodes, add the odd deg. nodes to either end (start & finiWsh at the odd degree nodes) -> Euler Path.

**4.b.iii**

Example solution:



**4.c.**

In any graph there are an even number of odd nodes.

Take a graph with 4 odd nodes (n1, n2, m1 and m2)

Assume there exists paths n1 -> n2 and m1 -> m2, and assume these paths are NOT arc-disjoint.

Therefore there are two nodes, say p and q, and the arc between them, p-q, which are part of both paths.

e.g.

However, p and q must be even (since there are only 4 odd nodes). At the moment they have 3 arcs incident to them, therefore to make the order even they both must have an extra arc incident. We must therefore have another path p -> q (which here I represent as a single extra arc p-q)

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CONTRADICTION: the paths n1 -> n2 and m1 -> m2 are now arc-disjoint.

Therefore in any graph with 4 odd nodes, we can pair up odd-degree nodes so that their paths are arc-disjoint.

We have only considered pairs of nodes at a time, so we can extend the principle to 6, 8 etc. odd nodes.

Therefore in ANY graph, we can pair up odd-degree nodes so that their paths are arc-disjoint.